JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 14, Number 4, Winter 2002

THE CONSERVED PENROSE-FIFE PHASE FIELD MODEL WITH SPECIAL HEAT FLUX LAWS AND MEMORY EFFECTS

ELISABETTA ROCCA

ABSTRACT. In this paper a phase-field model of Penrose-Fife type is considered for diffusive phase transitions with conserved order parameter. Different motivations lead to investigate the case when the heat flux is the superposition of two different contributions; one part is the gradient of a function of the absolute temperature ϑ, behaving like $1/\vartheta$ as ϑ approaches to 0 and like $-\vartheta$ as $\vartheta \nearrow +\infty$, while the other is given by the Gurtin-Pipkin law introduced in the theory of materials with thermal memory. An existence result for a related initial-boundary value problem is proven. Strengthening some assumptions on the data, the uniqueness of the solution is also achieved.

1. Introduction. This note is concerned with the study of the following initial-boundary value problem in the cylindrical domain $Q := \Omega \times (0,T)$, where $\Omega \subset \mathbf{R}^N$ $(N \le 3)$ is a bounded connected domain with a smooth boundary Γ and $T > 0$. Find a pair $(\vartheta, \chi) : Q \to \mathbf{R}^2$ satisfying

$$\partial_t\big(\vartheta + \lambda\chi\big) - \Delta\big(\psi(\vartheta) + k * \alpha(\vartheta)\big) = g \quad \text{in } Q, \tag{1.1}$$

$$-\,\partial_\nu\big(\psi(\vartheta) + k * \alpha(\vartheta)\big) = \gamma\big(\psi(\vartheta) + k * \alpha(\vartheta) - h\big) \quad \text{on } \Sigma := \Gamma \times (0,T), \tag{1.2}$$

$$\vartheta(\cdot, 0) = \vartheta^0 \quad \text{in } \Omega, \tag{1.3}$$

$$\partial_t\chi - \Delta\left(-\,\Delta\chi + \xi + \sigma'(\chi) + \frac{\lambda}{\vartheta}\right) = 0 \quad \text{in } Q, \tag{1.4}$$

$$\xi \in \beta(\chi), \quad \text{in } Q, \tag{1.5}$$

$$\partial_\nu\chi = 0, \quad \partial_\nu\left(-\,\Delta\chi + \xi + \sigma'(\chi) + \frac{\lambda}{\vartheta}\right) = 0 \quad \text{on } \Sigma, \tag{1.6}$$

$$\chi(\cdot, 0) = \chi^0 \quad \text{in } \Omega, \tag{1.7}$$

Received by the editors on December 10, 2001, and in revised form on July 1, 2002.