JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 14, Number 4, Winter 2002

SOLUTION OF VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS WITH GENERALIZED MITTAG-LEFFLER FUNCTION IN THE KERNELS

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ABSTRACT. The present paper is intended for the investigation of the integro-differential equation of the form

$$(*)\quad (\mathcal{D}_{a+}^{\alpha} y)(x) = \lambda \int_a^x (x-t)^{\mu-1} E_{\rho,\mu}^{\gamma}[\omega(x-t)^{\rho}] y(t)\,dt + f(x),$$
$$a < x \leqq b,$$

with complex $\alpha, \rho, \mu, \gamma$ and ω ($\mathrm{Re}\,(\alpha), \mathrm{Re}\,(\rho), \mathrm{Re}\,(\mu) > 0$) in the space of summable functions $L(a,b)$ on a finite interval $[a,b]$ of the real axis. Here $\mathcal{D}_{a+}^{\alpha}$ is the operator of the Riemann-Liouville fractional derivative of complex order α ($\mathrm{Re}\,(\alpha) > 0$) and $E_{\rho,\mu}^{\gamma}(z)$ is the function defined by

$$E_{\rho,\mu}^{\gamma}(z) = \sum_{k=0}^{\infty} \frac{(\gamma)_k}{\Gamma(\rho k + \mu)} \frac{z^k}{k!},$$

where, when $\gamma = 1$, $E_{\rho,\mu}^{1}(z)$ coincides with the classical Mittag-Leffler function $E_{\rho,\mu}(z)$, and in particular $E_{1,1}(z) = e^z$. Thus, when $f(x) \equiv 0$, $a = 0$, $\alpha = 1$, $\mu = 1$, $\gamma = 0$, $\rho = 1$, $\lambda = -i\pi g$, $\omega = i\nu$, g and ν are real numbers, the equation $(*)$ describes the unsaturated behavior of the free electron laser. The Cauchy-type problem for the above integro-differential equation is considered. It is proved that such a problem is equivalent to the Volterra integral equation of the second kind, and its solution in closed form is established. Special cases are investigated.

1. Introduction. It is well known that solutions of integro-differential equations of Volterra type can be obtained as solutions of

Received by the editors on June 21, 2002, and in revised form on November 14, 2002.
1991 AMS *Mathematics Subject Classification.* 45J05, 26A33, 33E12.
Key words and phrases. Integro-differential equation of Volterra type, Riemann-Liouville fractional integrals and derivatives, generalized Mittag-Leffler function, method of successive approximations.