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MULTI-LEVEL ITERATION METHODS FOR SOLVING INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. In this paper we develop multi-level iteration methods for solving Fredhom integral equations of the second kind based on the Galerkin method for which the Galerkin subspace has a multi-resolution decomposition. After expressing the equations using matrices of operators in accordance to the multi-resolution structure, we propose two iteration schemes for solving the equations that are analogues to the Jacobi and Gauss-Seidel iteration schemes for solving algebraic systems. We then discuss the two-grid nature of the schemes, compare them with the well-known two-grid schemes and a two-level scheme and prove their convergence. We also present our numerical implementation of these methods using piecewise linear polynomial wavelets for an integral equation with the logarithmic kernel.

1. Introduction. Numerical methods of solving integral equations often lead to solving very large linear algebraic systems that usually do not have convenient structures such as sparseness. Special efficient iteration methods (see, e.g., [**2, 3, 13, 14**]) are often used to solve such large systems by taking advantage of distinct properties from the integral equations and approximating subspaces. Recently more and more studies on using wavelet spaces for solving integral equations have emerged and shown promise ([**4, 6–9, 11, 12, 16, 17**]). The multi-resolution property of the wavelet subspaces makes such schemes most efficient in many applications, including solving integral equations. There have been existing iteration methods ([**5, 10**]) that utilize

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