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INTEGRAL OPERATORS OF MARCINKIEWICZ TYPE

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ABSTRACT. In this paper we study integral operators of Marcinkiewicz type. We formulate a general method which allows us to obtain the L^p boundedness of several classes of integral operators of Marcinkiewicz type. Our results extend as well as improve previously known results on Marcinkiewicz integral operators.

1. Introduction and statements of results. Let $n \geq 2$ and $\mathbf{S}^{n-1}$ be the unit sphere in $\mathbf{R}^n$ equipped with the normalized Lebesgue measure $d\sigma$. Suppose that Ω is a homogeneous function of degree zero on $\mathbf{R}^n$ that satisfies $\Omega \in L^1(\mathbf{S}^{n-1})$ and

$$\int_{\mathbf{S}^{n-1}} \Omega(x)\, d\sigma(x) = 0. \tag{1.1}$$

Let $\mathbf{U}(r)$ be the open ball centered at the origin in $\mathbf{R}^n$ with radius 2^r, $r \in \mathbf{R}$. If $r = \infty$, we shall let $\mathbf{U}(r) = \mathbf{R}^n$. For a suitable mapping $\Theta : \mathbf{U}(r) \to \mathbf{R}^d$, $d \in N$ and a measurable function $h : \mathbf{R}^+ \to \mathbf{R}$, let $\{\sigma_{t,\Theta,\Omega,h,r} : t \in \mathbf{R}\}$ be the family of measures defined on $\mathbf{R}^d$ by

$$\int_{\mathbf{R}^d} f\, d\sigma_{t,\Theta,\Omega,h,r} = 2^{-t}\chi_{(-\infty,r)}(t) \int_{|y|\leq 2^t} f(\Theta(y))|y|^{1-n}\Omega(y)h(|y|)\, dy, \tag{1.2}$$

where $\chi_{(-\infty,r)}(t)$ is the characteristic function of the interval $(-\infty, r)$. Define the operator $\mathbf{S}_{\Theta,\Omega,h,r}$ by

$$\mathbf{S}_{\Theta,\Omega,h,r}f(x) = \left(\int_{-\infty}^{\infty} |\sigma_{t,\Theta,\Omega,h,r} * f(x)|^2\, dt \right)^{1/2}. \tag{1.3}$$

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