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METRIC AND PROBABILISTIC INFORMATION ASSOCIATED WITH FREDHOLM INTEGRAL EQUATIONS OF THE FIRST KIND

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ABSTRACT. The problem of evaluating the information associated with Fredholm integral equations of the first kind, when the integral operator is self-adjoint and compact, is considered here. The data function is assumed to be perturbed *gently* by an additive noise so that it still belongs to the range of the operator. First we estimate upper and lower bounds for the ε-capacity (and then for the *metric information*), and explicit computations in some specific cases are given; then the problem is reformulated from a probabilistic viewpoint and use is made of the probabilistic information theory. The results obtained by these two approaches are then compared.

1. Introduction. Let us consider the following class of Fredholm integral equations of the first kind:

$$Af = g, \tag{1.1}$$

where $A : X \to Y$ is a self-adjoint compact operator, X and Y being the solution and the data space, respectively. Hereafter we set $X = Y = L^2[a,b]$.

Solving Equation (1.1) presents two problems:

a) The Range (A) is not closed in the data space Y. Therefore, given an arbitrary function $g \in Y$, it does not follow necessarily that there exists a solution $f \in X$.

b) Even if two data functions g_1 and g_2 belong to Range (A), and their distance in Y is small, nevertheless the distance between $A^{-1}g_1$ and $A^{-1}g_2$ can be unlimitedly large, in view of the fact that the inverse of the compact operator A is not bounded (X and Y being infinite dimensional space).

In the numerical applications, g is perturbed by a noise n which can represent either round-off numerical error or measurement error if g describes experimental data. Assuming in both cases that the