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BOUNDARY INTEGRAL EQUATIONS FOR THE BIHARMONIC DIRICHLET PROBLEM ON NONSMOOTH DOMAINS

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ABSTRACT. In this paper we study the boundary reduction of the biharmonic interior and exterior Dirichlet problems in a plane domain with piecewise smooth boundary. The mapping properties of single and double layer biharmonic potentials, of biharmonic boundary integral operators, the Calderon projections and Poincaré-Steklov operators for domain with corners are analyzed. We derive existence and uniqueness results for direct boundary integral equations, which are equivalent to the variational formulation of the problems.

1. Introduction. The paper is devoted to the direct boundary integral method for solving the interior and exterior Dirichlet problems of the biharmonic equation

$$\begin{aligned} \Delta^2 u = 0 \quad \text{in } \Omega \subset \mathbf{R}^2, \\ u|_\Gamma = f_1, \qquad \partial_n u|_\Gamma = f_2. \end{aligned} \tag{1.1}$$

Here Ω is an interior or exterior domain bounded by a closed piecewise smooth curve Γ with corners, and the Dirichlet data $(f_1, f_2) = (v|_\Gamma, \partial_n v|_\Gamma)$ are the traces of a function v belonging on a neighborhood of Γ to the Sobolev space H^2. For the exterior problem, one has to impose additionally a special behavior of the solution at infinity.

The aim of the present paper is the study of direct boundary integral formulations which are equivalent to the variational solution of (1.1). As the main result, we derive different systems of integral equations on Γ and describe their solvability conditions. To do so we introduce certain boundary integral operators for the bi-Laplacian and study mapping properties in the corresponding trace spaces of H^2-functions.

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