JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 11, Number 4, Winter 1999

CONVOLUTION CALCULUS FOR A CLASS OF SINGULAR VOLTERRA INTEGRAL EQUATIONS

KATSUNORI IWASAKI AND YUTAKA KAMIMURA

Dedicated to Professor Kazuo Okamoto on the occasion of his fiftieth birthday

ABSTRACT. For a class of singular Volterra integral equations we establish a necessary and sufficient condition for unique solvability in suitable function space settings. The discussion is based on the convolution calculus associated with the one-sided Mellin transform with weight 0. This study is motivated by some inverse nonlinear Sturm-Liouville problems, whose linearizations give rise to integral equations of our class. The method developed in this paper settles them in a unified manner.

1. Statement of main theorems. This paper is concerned with the integral equation for $u(x)$:

$$\int_0^1 \Phi(t)u(xt)\,dt = f(x), \quad a \le x \le b, \tag{1.1}$$

where $a \le 0 \le b$ and the kernel Φ and the right side f are known functions. Equation (1.1) can be rewritten as a Volterra integral equation of the first kind:

$$\int_0^x \Phi(s/x)u(s)\,ds = xf(x). \tag{1.2}$$

However, in general, this can not be handled by the standard method, see, e.g., [**2**, Chapter 2], [**6**, Section 3.3], [**10**, Section 40]. Indeed, the reduction to a Volterra integral equation of the second kind cannot be applied, since $(\partial/\partial x)\Phi(s/x)$ may have a singularity at $x = 0$. Also it

Received by the editors in revised form on September 24, 1998.
1991 AMS *Mathematics Subject Classification.* 45D05, 45E10, 44A35.
Key words and phrases. Convolution calculus, singular Volterra integral equations.
This research was partially supported by Grant-in-Aid for Scientific Research No. 09640157 and No. 07640179, the Ministry of Education, Science, Culture and Sports, Japan.