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INITIAL VALUE PROBLEMS FOR NONLINEAR SECOND ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS IN BANACH SPACES

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ABSTRACT. In this paper the author uses the fixed point theory to investigate the existence of solutions of initial value problems for nonlinear second order impulsive integro-differential equations in Banach spaces.

1. Introduction. The theory of impulsive differential equations has become an important area of investigation in recent years, see [**5**]. In Section 4.3 of [**4**] and [**1**], the authors discussed the existence of solutions of boundary value problems for nonlinear second order impulsive integro-differential equations in Banach spaces E by means of Darbo's fixed point theorem. Now, under more wide conditions, see Remark 2, this paper shall also use fixed point theory to investigate the existence of solutions of initial value problems (IVP) for second order impulsive integro-differential equations in E. But we cannot obtain the results in this paper directly by means of Darbo's fixed point theorem used in [**4**] and [**1**].

Consider the IVP for impulsive integro-differential equations in a Banach space E:

$$\begin{aligned} x'' &= f(t,x,x',Tx,Sx), \quad t\in J, \quad t\neq t_k,\\ \Delta x|_{t=t_k} &= I_k(x(t_k),x'(t_k)),\\ \Delta x'|_{t=t_k} &= \bar{I}_k(x(t_k),x'(t_k)), \quad k=1,2,\dots,m,\\ x(0)&=x_0, \qquad x'(0)=x_1, \end{aligned} \tag{1.1}$$

where $f\in C[J\times E\times E\times E\times E, E]$, $J=[0,a](a>0)$, $0<t_1<t_2\cdots<t_m<a$, $I_k,\bar{I}_k\in C[E\times E,E]$, $x_0,x_1\in E$ and

$$(Tx)(t)=\int_0^t k(t,s)x(s)\,ds, (Sx)(t)=\int_0^a h(t,s)x(s)\,ds, \tag{1.2}$$

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