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CONSTRUCTIVE ANALYSIS OF PURELY INTEGRAL BOLTZMANN MODELS

ALVISE SOMMARIVA AND MARCO VIANELLO

ABSTRACT. Existence and uniqueness of nonnegative L^1 stationary solutions of the space homogeneous force-free Boltzmann equation are proved. Classical assumptions are weakened, and certain unphysical restrictions are removed. The proof is constructive, being based on Picard iterations for an equivalent Hammerstein type formulation, within the theory of decreasing operators in ordered function spaces.

1. Introduction. Within the so-called "scattering kernel" formulation of the Boltzmann equation [**4**], modeling diffusion of particles in a mixture of two species (field particles and test particles), study of the *space homogeneous forceless* case leads naturally to the nonlinear integral equation

$$N\hat{g}_r(|\mathbf{v}|)f(\mathbf{v}) + f(\mathbf{v})\int_{\mathbf{R}^3} g_r(|\mathbf{v}-\mathbf{v}'|)f(\mathbf{v}')\,d\mathbf{v}' = Q_0S(\mathbf{v}), \quad \mathbf{v}\in\mathbf{R}^3, \tag{1}$$

which amounts to the search for *stationary* solutions, cf., eg., [**1** and references therein]. In (1) $\hat{g}_r$ and g_r describe the removal effects between particles, N is the fixed total density of the field particles, and $Q_0S(\mathbf{v})$ represents the constant and space uniform emission rate of the test particles, with $\int_{\mathbf{R}^3} S(\mathbf{v})\,d\mathbf{v} = 1$.

As pointed out in [**1**], equation (1) provides a model for physical situations where removal of test particles between themselves is a dominant event, like, for instance, in the study of chemical and biological

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