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OPERATOR NORMS OF POWERS OF THE VOLTERRA OPERATOR

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1. Introduction. The Volterra operator $V : L^2[0,1] \to L^2[0,1]$ will be defined by

$$Vf(x) = \int_0^x f(t)\,dt, \tag{1.1}$$

where f is real valued function.

Definition 1.1. The *operator norm*, $\|.\|$, is defined by

$$\|T\| = \sup_{\|f\|_2=1} \|Tf\|_2, \tag{1.2}$$

where

$$\|f\|_2 = \left[\int_0^1 |f(t)|^2\ dt\right]^{1/2}. \tag{1.3}$$

It is not difficult to show that the operator norm of V is $2/\pi$. In [**5**] N. Lao and R. Whitley give the numerical evidence which led them to the conjecture that

$$\lim_{m\to\infty} \|m!V^m\| = 1/2. \tag{1.4}$$

The purpose of this article is to verify that this is indeed the case. The analysis will be presented for a more general operator defined as follows.

Definition 1.2. The linear operator $A : L^2[0,1] \to L^2[0,1]$ is given by

$$Af(x) = \int_0^x a(x-t)f(t)\,dt, \tag{1.5}$$

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