

**EXISTENCE OF SOLUTIONS FOR A CLASS
OF INTEGRODIFFERENTIAL EQUATIONS
IN BANACH SPACES**

FANGQI CHEN

ABSTRACT. An existence theorem of solutions for a class of nonlinear integrodifferential equations in Banach spaces is established. This is achieved by means of the Mönch fixed point theorem and an integration inequality for the measure of noncompactness.

1. Introduction. Let E be a real Banach space, $R^+ = \{t \in R^1 : t \geq 0\}$. Consider the IVP of nonlinear integrodifferential equations on the infinite interval R^+ in Banach space E ,

$$(1) \quad x'(t) = F\left(t, x(t), \int_0^t K(t, s, x(s)) ds\right), \quad x(0) = 0, \quad t \in R^+$$

where $F \in C[R^+ \times E \times E, E]$, $K \in C[R^+ \times R^+ \times E, E]$. In the case that IVP (1) is a scalar integrodifferential equation, the existence theorem of solutions has been obtained by means of the topological transversality arguments in [1]. But it is easy to see that the method used in [1] is not successful in the Banach space case. In this paper we shall use the Mönch fixed point theorem and an integration inequality for the measure of noncompactness to investigate the existence of solutions of IVP (1). An existence theorem is obtained.

2. Preliminaries. Throughout this paper, for $T > 0$, $C[[0, T], E]$ denotes the Banach space with supremum norm. For $D \subset C[[0, T], E]$, we write $D(t) = \{x(t) : x \in D\} \subset E$, $t \in [0, T]$. α denotes the Kuratowski measure of noncompactness.

Received by the editors on July 23, 1997, and in revised form on April 25, 1998.
This project supported by the National Natural Science Foundation of China
and the State Education Commission Doctoral Foundation of China.

Copyright ©1999 Rocky Mountain Mathematics Consortium