JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 11, Number 3, Fall 1999

A NEW INTEGRAL EQUATION FORMULATION FOR THE SCATTERING OF PLANE ELASTIC WAVES BY DIFFRACTION GRATINGS

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ABSTRACT. The scattering of plane elastic waves by a rigid periodic surface is considered. The Green's tensor for a half-space with a rigid surface is introduced and its properties, notably its asymptotic decay rate in horizontal layers above the plane, are analyzed. The Green's tensor is then used to define single and double layer potentials for a periodic surface, making use of the generalized stress tensor. Subsequently, a novel integral equation formulation for the scattering of plane waves by a diffraction grating is derived using a Brakhage/Werner type ansatz for the solution. Employing the Fredholm alternative, existence of solution is proved for all angles of incidence and all wave-numbers.

1. Introduction. It is the object of this paper to derive a new integral equation formulation for a certain scattering problem, namely the scattering of a time harmonic plane elastic wave by an unbounded periodic structure. The propagation of time harmonic waves with circular frequency ω in an elastic solid with Lamé constants μ, λ $(\mu, \lambda+\mu>0)$ and density ρ is governed by Hooke's law

$$\tau_{jk} = \lambda \operatorname{div} \mathbf{u} \delta_{jk} + \mu\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}\right), \quad j,k = 1,2,3, \tag{1}$$

and by the equation of motion

$$\sum_{k=1}^{3} \frac{\partial \tau_{jk}}{\partial x_k} + \omega^2 \rho u_j = 0, \quad j = 1,2,3. \tag{2}$$

Here the vector field $\mathbf{u}$ denotes the displacements and τ the stress tensor. We will assume that the density ρ is constant throughout the

Received by the editors on November 2, 1998, and in revised form on February 16, 1999.