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FAST COLLOCATION SOLVERS FOR INTEGRAL EQUATIONS ON OPEN ARCS

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ABSTRACT. In this work we develop a unified approach for numerical approximation and fast solution of classical integral equations on open arcs. The approximation is obtained applying the cosine transform and fully discrete trigonometric collocation together with an asymptotic approximation of the operator. The computed approximation is of optimal accuracy order in a large scale of Sobolev norms, and it can be obtained in $\mathcal{O}(n \log n)$ arithmetical operations. Our results cover logarithmic singular integral equations, Cauchy singular integral equations, as well as hypersingular integral equations.

1. Introduction. In many applications the boundary integral method leads to solution of an integral equation on an open arc, when two-dimensional phenomena are considered. In the basic examples the arising integral equations can be covered by the following types: logarithmic singular integral equations, Cauchy singular integral equations and hypersingular integral equations. For the parametrized forms of the model equations see (2.1)–(2.3). Equations of these types come from various fields such as fracture mechanics, aerodynamics, electromagnetism and elasticity, for example. Except for some special cases the arising integral equation cannot be solved explicitly but requires an approximate solution by numerical methods. In the literature various numerical schemes have been proposed for particular examples. These schemes cover the standard spline based methods [**31, 8, 38, 12**] as well as trigonometric methods and polynomial approximation with their fully discrete variants including also other quadrature methods [**17, 2, 9, 25, 5, 19, 6, 7, 11**]. This list is not complete, in particular for Cauchy singular equations there are many earlier studies but they can be traced from the works mentioned here. Also fast solution has been considered: for the case of Cauchy singular equation with a polynomial approximation, see [**3, 4**].

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