JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 11, Number 1, Spring 1999

ON THE NUMERICAL SOLUTION OF LINEAR EVOLUTION PROBLEMS WITH AN INTEGRAL OPERATOR COEFFICIENT

I. GAVRILYUK, V. MAKAROV AND R. CHAPKO

ABSTRACT. We present a method for the numerical solution of first order nonstationary problems with a pseudodifferential operator coefficient on a manifold. Using the Cayley transform we get an explicit representation of the exact solution and reduce the problem to a sequence of stationary equations which then are transformed into hypersingular integral equations of the second kind. For the numerical solution of these integral equations we use a collocation procedure based on appropriate trigonometric interpolation quadratures. Using the numerical solution of the integral equations and the explicit representation of the exact soution, we get a fully discrete approximation with respect to time and to spatial discretization parameters. In the case of a circle, the analysis of convergence and error estimates are given which show automatic dependence of the error order on the smoothness of the exact solution (the spectral property with respect to both the time and the spatial discretization parameters).

1. Introduction. Differential equations of the type

$$\frac{du}{dt} + \tilde{A}u = 0, \qquad u(0) = u_0 \tag{1.1}$$

where $u : \mathbf{R}_+ \to E$ is a vector-valued function with values in a Banach space E and $\tilde{A} : E \to E$ is an operator coefficient, describe many physical and technical processes. For example, problems of the type (1.1) arise in the theory of water and gravity waves, in the theory of viscous flows, in acoustics and elastostatics, where E is a Banach space of functions defined on a piecewise smooth curve and

Received by the editors on December 17, 1997, and in revised form on February 12, 1998.
Key words and phrases. Cayley transform, explicit representation, collocation methods, spectral property, Toeplitz matrix, circulant matrix, Fast Fourier Transform.
AMS *Mathematics Subject Classification.* 45L10, 65R20, 65J10, 65M70.
Research of the first and second authors partly supported by DFG and DAAD.
Research of the third author partly supported by KAAD.