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# THE DISCRETE PETROV-GALERKIN METHOD FOR WEAKLY SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. We propose discrete Petrov-Galerkin methods for Fredholm integral equations of the second kind with weakly singular kernels. To study the convergence of these methods, we develop a theoretical framework for analysis of a large class of numerical schemes including the discrete Galerkin, Petrov-Galerkin, collocation methods and quadrature methods. The theory is then applied to establish convergence results of the discrete Petrov-Galerkin methods. We also suggest a discrete iterated Petrov-Galerkin approximation for the solutions of these equations and prove a superconvergence property when the kernels are assumed to be smooth. Numerical examples are presented to illustrate the theoretical estimate for the error of approximation of these methods.

**1. Introduction.** We begin our presentation with a brief review of the literature. The Petrov-Galerkin method and the iterated Petrov-Galerkin method for Fredholm integral equations of the second kind were studied in [**7**], where the notions of the generalized best approximation and the regular pair of a trial space sequence and a test space sequence were developed to serve as an approach for the analysis of the methods. Several specific constructions of the Petrov-Galerkin elements in 1-D and 2-D were also designed in the paper. A general construction of the univariate Petrov-Galerkin elements of the piecewise polynomials were proposed in [**6**] and it was used to develop wavelet Petrov-Galerkin methods for integral equations of one dimension. Some early work on the Petrov-Galerkin method was found in [**10**]. Several Petrov-Galerkin elements constructed in [**7**] and [**6**] were proved to be useful in numerical solutions of integral equations.

Before describing the discrete Petrov-Galerkin method, a few remarks are in order on a comparison between the standard Galerkin method

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