

NUMERICAL CONFORMAL MAPPING
FOR EXTERIOR REGIONS VIA THE
KERZMAN-STEIN KERNEL

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*Dedicated to Philip M. Anselone, in appreciation of a
long friendship and lasting contributions*

ABSTRACT. A simple numerical method is described for computing a conformal map from a domain exterior to a smooth Jordan curve in the complex plane onto the exterior of the unit disk. The numerical method is based on a boundary integral equation similar to the Kerzman-Stein integral equation for interior mappings. Typical examples show that numerical results of high accuracy can be obtained provided that the boundaries are smooth.

1. Introduction. Let Γ be a Jordan curve, Ω^+ and Ω^- be the interior and exterior of Γ , respectively. It is well known, see, e.g., [7, p. 374], that there exists a unique function \tilde{R} , called the *exterior mapping function* of Ω , which maps Ω^- onto the exterior of the unit disk such that $\tilde{R}(\infty) := \lim_{z \rightarrow \infty} \tilde{R}(z) = \infty$ and the Laurent series of \tilde{R} at ∞ has the form

$$(1.1) \quad \tilde{R}(z) = \gamma^{-1}z + a_0 + a_1z^{-1} + \cdots, \quad \gamma > 0, z \in \Omega^-.$$

The positive constant γ is called the *capacity* of Γ , or the *transfinite diameter* of Ω^+ .

Since Γ is a Jordan curve, \tilde{R} can be extended to a homeomorphism of the closure $\Omega^- \cup \Gamma$ to the closure $|w| \geq 1$. Thus as $z(t)$ traverses along Γ , the image point $\tilde{R}(z(t))$ describes the unit circle. Hence

$$(1.2) \quad \tilde{R}(z(t)) = e^{i\theta(t)}, \quad 0 \leq t \leq \beta,$$

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