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A WAVELET ALGORITHM FOR THE SOLUTION OF A SINGULAR INTEGRAL EQUATION OVER A SMOOTH TWO-DIMENSIONAL MANIFOLD

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ABSTRACT. In this paper we consider a piecewise bilinear collocation method for the solution of a singular integral equation over a smooth surface. Using a fixed set of parametrizations, we introduce special wavelet bases for the spaces of test and trial functions. The trial wavelets have two vanishing moments only if their supports do not intersect the lines belonging to the common boundary of two subsurfaces defined by different parameter representations. Nevertheless, analogously to well-known results on wavelet algorithms, the stiffness matrices with respect to these bases can be compressed to sparse matrices such that the iterative solution of the matrix equations becomes fast. Finally we present a fast quadrature algorithm for the computation of the compressed stiffness matrix.

1. Introduction. It is a well-known fact that usual finite element discretizations of linear integral equations, e.g., of boundary integral equations, lead to systems of linear equations with fully populated matrices. Thus, even an iterative solution method requires a huge number of arithmetic operations and a large storage capacity. In order to improve these finite element approaches, several new algorithms have been developed. For a relatively wide class of boundary integral equations, Rokhlin and Greengard [**37, 20**] have introduced their methods of multipole expansion, Hackbusch and Nowak [**21**], cf. also [**38**], have considered panel clustering algorithms, and Brandt and Lubrecht [**3**] have set up multilevel schemes. Another approach for saving storage and computation time consists in employing wavelet bases of the finite element spaces. This idea goes back to Beylkin, Coifman and Rokhlin [**2**] and has been thoroughly investigated by Dahmen, Petersdorff, Prößdorf, Schneider and Schwab [**13, 14, 12, 15, 32, 31, 30, 39**], cf. also the contributions by Alpert, Harten, Yad-Shalom, Dorobantu, Kleemann and the author [**1, 22, 19, 9, 10,**

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