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HIGHER ACCURACY METHODS FOR SECOND-KIND VOLTERRA INTEGRAL EQUATIONS BASED ON ASYMPTOTIC EXPANSIONS OF ITERATED GALERKIN METHODS

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Dedicated to Professor Phil Anselone, with our best wishes

ABSTRACT. On the basis of asymptotic expansions, we study the Richardson extrapolation method and two defect correction schemes by an interpolation post-processing technique, namely, interpolation correction and iterative correction for the numerical solution of a Volterra integral equation by iterated finite element methods. These schemes are of higher accuracy than the postprocessing method and analyzed in a recent paper [**5**] by Brunner, Q. Lin and N. Yan. Moreover, we give a positive answer to a conjecture in [**5**].

1. Introduction. In this paper we are concerned with finite element methods for the Volterra integral equation of the second kind,

$$y(t) = g(t) + \int_0^t K(t,s)y(s)\,ds, \quad t \in I := [0,1], \tag{1.1}$$

where $g : I \to \mathbf{R}$ and $K : D \to \mathbf{R}$ (with $D := \{(t,s) : 0 \le s \le t \le 1\}$) denote given (continuous) functions. It is well known that if $K \in C^m(D)$ and $g \in C^m(I)$, the solution y of (1.1) is in $C^m(I)$.

The study of (local) superconvergence properties of collocation methods for Volterra integral equations (1.1) (as well as for second-kind Fredholm integral equations) and of methods for accelerating the convergence orders has received considerable attention since the early 1980s, compare, for example, [**1, 2, 4, 8, 9** and **13**].

In this note we present two defect correction schemes, namely, interpolation correction and iterative correction, for the numerical solution

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