JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 10, Number 3, Fall 1998

ABSTRACT VOLTERRA EQUATIONS OF THE SECOND KIND

MARTIN VÄTH

Dedicated to Professor Philip M. Anselone

ABSTRACT. We consider the equation $x = Vx + f$ with a nonlinear Volterra operator V in a large class of spaces. We prove that it has a local solution if V is continuous and compact or (in case of a regular space) condensing.

We study the connection of local and global solutions and gain in particular an abstract extension principle and some results on global solutions for a 'nonlinear Fredholm' case and for the case of positively homogeneous operators.

Moreover, we show that compact linear Volterra operators in (not necessarily regular) ideal spaces have spectral radius zero, which generalizes a result of Zabrejko.

The algebraic definition of 'Volterra operator' in the article matches a much wider class of operators than the classical Volterra operators. This more general notion leads to new results, even when applied to the classical linear Volterra operator in L_p.

We also give some applications to differential equations in Banach spaces.

0. Introduction. We are concerned with the Volterra equation of the second kind, $x = Vx+f$ with a nonlinear Volterra operator V. Here the term *Volterra operator* is defined in an abstract way: in Section 1 we give the precise definition. Although this technical definition is of a purely algebraic nature, it 'catches' the typical behavior of Volterra operators. In Sections 2 and 3 we study the existence of 'local,' respectively 'global,' solutions of the Volterra equation. Since the results in these sections are rather involved, we summarize the most

---

Received by the editors on February 25, 1997, and in revised form on September 8, 1997.
1991 AMS *Mathematics Subject Classification.* Primary 45D05, Secondary 45N05, 34G20, 47H15.
Key words and phrases. Nonlinear Volterra equation, abstract Volterra operator, differential equations in Banach spaces, integral equations in Banach spaces.
This paper was written in the framework of a DFG project (Ap 40/6-2). Financial support by the DFG is gratefully acknowledged.