

A SINC QUADRATURE METHOD FOR THE DOUBLE-LAYER INTEGRAL EQUATION IN PLANAR DOMAINS WITH CORNERS

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ABSTRACT. A convergence and error analysis is given for a Nyström method on a graded mesh based on sinc quadrature for an integral equation of the second kind with a Mellin type singularity. An application to the double-layer integral equation for planar domains with corners is described.

1. Introduction. Sinc approximation methods have been successfully employed for problems where the solution has singularities, for example for partial differential equations and associated integral equations in domains with corners (see [8, Chapter 5], and [12, Sections 6.5, 6.6, and 7.4]). Given this success, we felt a need to explain theoretically the numerical performance by an error and convergence analysis for a particular situation. For this we have chosen the application of a sinc quadrature method for the solution of the integral equation of the second kind arising from the double-layer approach to solve the Dirichlet problem for the Laplace equation in planar domains with corners. Since the solution of the integral equation develops a singularity of the derivatives at the corner, in the discretization of this integral equation a graded mesh must be used in order to achieve a satisfactory accuracy. Quadrature or Nyström methods for the double-layer integral equation using graded meshes have been previously considered by Graham and Chandler [4], Atkinson and Graham [2], Kress [7], Jeon [5] and Elliott and Prössdorf [3].

Because of the Mellin type singularity of the double-layer kernel for domains with corners, the double-layer integral operator is no longer compact in the space of continuous functions. Therefore the Riesz theory cannot be immediately employed for establishing existence of a solution. Following the classical work of Radon [11], this difficulty can be remedied by splitting the operator into an operator with norm less than one, reflecting the singular behavior at the corner, and a compact operator (see also [6, p. 76]). It appears natural that for a convergence and error analysis for corresponding quadrature