

ABSTRACT HYPERBOLIC VOLTERRA INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT. This paper is devoted to the study of the problem of global solvability for the abstract hyperbolic Volterra integrodifferential equation

$$(VIE) \quad \begin{cases} u'(t) = A(t)u(t) + \int_0^t g(t, s, u(s)) ds + f(t) & \text{for } t \geq 0 \\ S(t)u(t) \in D & \text{for } t \geq 0 \\ u(0) = \phi \end{cases}$$

in a general Banach space X . The result obtained here is applicable to semilinear hyperbolic integrodifferential equations with the so-called third kind boundary conditions in a space of continuous functions.

0. Introduction. The main object of this paper is the study of global solvability for the semilinear hyperbolic Volterra integrodifferential equation

$$(VIE) \quad \begin{cases} u'(t) = A(t)u(t) + \int_0^t g(t, s, u(s)) ds + f(t) & \text{for } t \geq 0 \\ S(t)u(t) \in D & \text{for } t \geq 0 \\ u(0) = \phi \end{cases}$$

in a general Banach space X . Here $\{A(t) : t \geq 0\}$ is a given family of bounded linear operators on Y to X , where Y is another Banach space continuously imbedded in X , D is a closed linear subspace in Y , $\{S(t) : t \geq 0\}$ is a given family of isomorphisms of X onto X , $g(t, s, w)$ is an X -valued function of $(t, s) \in \Delta := \{(t, s) : 0 \leq s \leq t < \infty\}$ and $w \in Y$, and $f \in C^1([0, \infty) : X)$.

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