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EXISTENCE AND STABILIZATION OF SOLUTIONS TO THE PHASE-FIELD MODEL WITH MEMORY

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ABSTRACT. A phase field model is considered when the classical Fourier law is replaced by the linearized Gurtin-Pipkin constitutive assumption for the heat flux. The resulting system of partial differential equations consists in a Volterra integro-differential equation coupled with a nonlinear parabolic inclusion. The initial and boundary value problem with homogeneous Neumann boundary conditions is investigated for a kernel of positive type. Results on the long-time behavior of solutions are obtained in a quite general setting.

1. Introduction. This paper is devoted to the study of the so-called phase-field model, see, e.g., [**6, 10, 12**], for the temperature ϑ and the phase variable χ, in the case where the classical Fourier law $\mathbf{q} = -k_0 \nabla\vartheta$, k_0 constant, is replaced by the following nonlocal condition

$$\mathbf{q}(x,t) = -\int_{-\infty}^{t} k(t-s)\ \nabla\vartheta(x,s)\,ds, \tag{1.1}$$

for a kernel $k : (0,+\infty) \to \mathbf{R}$ of positive type. Here, $x \in \Omega$ denotes the space variable and t represents the time, letting Ω be a bounded domain of $\mathbf{R}^3$ with smooth boundary Γ and t vary in $(0,+\infty)$, as the past evolution of ϑ is supposed to be a known function ϑ_P up to $t = 0$,

$$\vartheta = \vartheta_P \quad \text{in } \Omega \times (-\infty, 0). \tag{1.2}$$

The relation (1.1) then states that the heat flux depends only on the temporal history of the temperature gradient and turns out to be compatible with classical thermodynamical laws whenever k is a kernel of positive type, cf. [**11**]. This is indeed a basic assumption in our approach. Two other important facts to be mentioned at once are the facts that we allow quadratic nonlinearities in the model and that we consider an initial-boundary value problem with homogeneous Neumann boundary conditions for both unknowns. Concerning the

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