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BPX PRECONDITIONER FOR HYPERSINGULAR INTEGRAL EQUATIONS

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ABSTRACT. In this paper we present the BPX (Bramble, Pasciak and Xu) preconditioner method for the Galerkin approximation of hypersingular integral equations on the interval $\Gamma = (-1, 1)$. The condition number of the resulting matrix with respect to the BPX preconditioner is shown to behave like $\mathcal{O}(h^{-\varepsilon})$ where ε is small and depends on the singularity of the exact solution at the end points of the open curve Γ. When Γ is closed, ε is reduced to zero, hence the condition number is independent of the mesh size. The implementations are based on the preconditioned conjugate gradient method using the BPX preconditioner. The numerical results are presented with a comparison between BPX preconditioner and HB (hierarchical basis) preconditioner.

1. Introduction. The discretization of partial differential equations and boundary integral equations leads to very large systems of linear equations, the direct solution of which can be very expensive in terms of storage and computational work. We now consider the BPX preconditioner method developed in the 1990s for finite element methods. Together with multigrid methods [**2**], domain decomposition methods [**9**], and hierarchical basis methods [**17**], the BPX preconditioner method is the fastest known method for solving large systems of linear equations arising from the discretization of partial differential equations. The theoretical foundation of the BPX preconditioner method started with Bramble et al. [**4**]. The BPX preconditioner method usually needs slightly more iteration steps than the multigrid methods, but the higher flexibility of these algorithms simplifies the use of parallel computing (the single subspace corrections are not applied in a sequential order but in parallel, see (3.8)). Another advantage is that it allows a simpler, more natural data structure and is therefore much better for non-uniformly refined grids. Consequently, it is possible to combine this method with adaptive methods. The combined adaptive

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