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TOLERANT QUALOCATION—A QUALOCATION METHOD FOR BOUNDARY INTEGRAL EQUATIONS WITH REDUCED REGULARITY REQUIREMENT

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Dedicated to Professor Wolfgang L. Wendland on the occasion of his 60th birthday.

ABSTRACT. We study a modification of the qualocation method for boundary integral equations on smooth curves, which allows the same high-order convergence as the original qualocation method but with reduced smoothness assumptions on the exact solution. This 'tolerant qualocation' method differs from the original one only in that the exact inner product is used on the righthand side of the qualocation equation, whereas in the original method a specially designed approximate inner product is used on both sides. The modified method achieves exactly the same error estimates as the Petrov-Galerkin method, at greatly reduced cost.

1. Introduction. We present in this paper a modification of the qualocation method introduced in [**11, 15, 3**], which allows the same high-order convergence (in an appropriate negative norm), but with reduced smoothness assumptions on the exact solution. 'Tolerant qualocation' seems an appropriate name for this modification, as a reminder of its forgiving nature. As in [**3**], the problem studied is

$$Lu = f,$$

where L is a pseudodifferential operator, so that the equation represents a boundary integral equation on a smooth curve.

For a review of the qualocation method, we refer the reader to [**12**]. We briefly recall here that the qualocation method is a semi-discrete Petrov-Galerkin method in which the outer inner product is replaced by a specially chosen quadrature rule. The same quadrature rule is used in the inner product on the right side of the equation. That

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