JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 10, Number 1, Spring 1998

# SEMI-DISCRETE FINITE ELEMENT APPROXIMATIONS FOR LINEAR PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS WITH INTEGRABLE KERNELS

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ABSTRACT. In this paper we consider finite element methods for general parabolic integro-differential equations with integrable kernels. A new approach is taken, which allows us to derive optimal $L^p$, $2 \leq p \leq \infty$, error estimates and superconvergence. The main advantage of our method is that the semi-discrete finite element approximations for linear equations, with both smooth and integrable kernels, can be treated in the same way without the introduction of the Ritz-Volterra projection; therefore, one can make full use of the results of finite element approximations for elliptic problems.

**1. Introduction.** In this paper we study numerical solutions by finite element methods for the following parabolic integro-differential equation:

$$\begin{cases} u_t + Au = \int_0^t a(t-s)Bu(s)\,ds + f(t) & \text{in } \Omega \times J, \\ u = 0 & \text{on } \partial\Omega \times J, \\ u(\cdot, 0) = v & \text{on } \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^d$, $d \geq 1$, is a bounded domain with smooth boundary $\partial\Omega$, $J = (0, T_0]$, $T_0 > 0$, $a(t) \in L^1(J)$ an integrable kernel, $f$ and $v$ are known smooth functions. $A$ is a positive definite second order elliptic operator,

$$A(t) = -\sum_{i,j=1}^{d} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial}{\partial x_j}\right) + a(x)I, \quad a(x) \geq 0,$$

$$a_{ij}(x) = a_{ji}(x), \quad i,j = 1, \ldots, d,$$

$$\sum_{i,j=1}^{d} a_{ij}\xi_i\xi_j \geq C_0 \sum_{i=1}^{d} \xi_i^2, \quad C_0 > 0,$$

Received by the editors on June 15, 1995, and in revised form on June 1, 1997.
AMS (MOS) *Mathematics Subject Classification.* 65N30, 45K05.
*Key words and phrases.* Integrable kernel, finite element, error estimates, maximum norm, superconvergence, parabolic, integro-differential.
This work is supported in part by NSERC (Canada).