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THE FINITE-SECTION APPROXIMATION FOR ILL-POSED INTEGRAL EQUATIONS ON THE HALF-LINE

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ABSTRACT. Integral equations on the half-line are commonly approximated by the finite-section approximation, in which the infinite upper limit is replaced by a positive number called the finite-section parameter. In this paper we consider the finite-section approximation for the first kind integral equations, which are typically ill-posed and call for regularization. For some classes of such equations corresponding to inverse problems from optics and astronomy, we indicate the finite-section parameters that allow us to apply standard regularization techniques. Two discretization schemes for the finite-section equations are also proposed and their efficiency is studied.

1. Introduction. In this paper we consider integral equations of the form

$$Kx(t) := \int_0^\infty k(t,\tau)b(\tau)x(\tau)\,d\tau = y(t), \quad t \geq 0, \tag{1.1}$$

under the assumptions that $x(t),\ y(t) \in L_2(0,\infty)$, and $k(t,\tau),\ b(\tau)$ are continuous functions such that for $t,\tau \to \infty$ $|k(t,\tau)| \sim (t\tau)^{-\kappa}$, $|b(\tau)| \sim \tau^\beta$, κ, $\beta > 0$. More precisely, we assume that there are the constants c_k, c_b such that, for any $t,\tau \in [0,\infty)$,

$$|k(t,\tau)| \leq \frac{c_k}{[(1+t)(1+\tau)]^\kappa}, \quad |b(\tau)| \leq c_b\tau^\beta. \tag{1.2}$$

Example. Many inverse problems in optics and astronomy can be modeled, at least approximately, by the problem of solving an integral equation of the type (1.1) and (1.2). An example is an equation which determines the particle size distribution of spherical particles by scattering methods (cf. [**2, 4, 5**]) which is given by

$$\int_0^\infty x(\tau)\tau^4 \left[\frac{2J_1(t\tau)}{t\tau}\right]^2 d\tau = y(t), \tag{1.3}$$