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RIGOROUS RESULTS ON THE ASYMPTOTIC SOLUTIONS OF SINGULARLY PERTURBED NONLINEAR VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. This paper studies singularly perturbed Volterra integral equations of the form

$$\varepsilon u(t) = f(t;\varepsilon) + \int_0^t g(t,s,u(s))\,ds, \quad 0 \le t \le T,$$

where ε is a small parameter. The function $f(t;\varepsilon)$ is defined for $0 \le t \le T$ and $g(t,s,u)$ for $0 \le s \le t \le T$. There are many existence and uniqueness results known that ensure that a unique continuous solution $u(t;\varepsilon)$ exists for all small $\varepsilon > 0$. The aim is to find asymptotic approximations to these solutions and rigorously prove the asymptotic correctness. This work is restricted to problems where there is an *initial layer*; various hypotheses are placed on g to exclude other behaviors.

1. Introduction. A singular perturbation problem is a problem which depends on a parameter (or parameters) in such a way that solutions of the problem behave nonuniformly as the parameter tends toward some limiting value of interest. The nature of the nonuniformity of the solutions can vary. This article concerns solutions of nonlinear Volterra integral equations in which such nonuniformity occurs in an isolated narrow region called the initial (or boundary) layer. The thickness of the layer vanishes as the parameter tends to zero.

In particular, we consider the nonlinear singularly perturbed Volterra integral equation

$$\varepsilon u(t) = f(t;\varepsilon) + \int_0^t g(t,s,u(s))\,ds, \quad 0 \le t \le T, \tag{1.1}$$

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