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BOUNDEDNESS OF THE GLOBAL ERROR OF SOME LINEAR AND NONLINEAR METHODS FOR VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. The boundedness of the global error of Runge-Kutta and direct quadrature methods for nonconvolution linear systems of Volterra integral equations of the second kind is analyzed.

1. Introduction. In this paper we are interested in the boundedness of the global error of Volterra Runge-Kutta (VRK) and direct quadrature (DQ) methods for linear Volterra integral equations (VIEs) of the type

$$y(t) = g(t) + \int_0^t k(t,s)y(s)\,ds, \quad t \in [0,T] \qquad (1.1)$$
$$y,\, g \in R^d, \quad k(t,s) \in R^{d\times d}.$$

Because of the hereditary character of the problem (1.1), any numerical methods applied to it give rise to a Volterra Discrete Equation (VDE) which is a difference equation with unbounded order. This makes the analysis of the behavior of the global error of numerical methods for Volterra problems a very involved task.

The behavior of the global error is strictly connected with the stability of numerical methods. In numerical analysis the necessity of studying stability arises anytime one is faced with a general step by step method for computing a sequence of values (for example numerical methods for ODEs or VIEs) and the name numerical stability is used with several different meanings. We could summarize the definitions of stability, both for numerical methods for ODEs and VIEs, in two concepts which, in some cases, coincide. The first concept is the basis of the weak stability theory for numerical methods for ODEs (see [**11**,

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