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UPPER AND LOWER BOUNDS FOR SOLUTIONS OF NONLINEAR VOLTERRA CONVOLUTION INTEGRAL EQUATIONS WITH POWER NONLINEARITY

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ABSTRACT. The Volterra nonlinear integral equation

$$\varphi^m(x) = a(x)\int_0^x k(x-t)b(t)\varphi(t)\,dt + f(x), \quad 0 < x < d \leq \infty$$

with $m > 1$ and real nonnegative functions $a(x)$, $k(u)$, $b(t)$ and $f(x)$ is studied. In the general case some upper bounds of the average

$$\frac{1}{x}\int_0^x \varphi(t)\,dt$$

of the solution are given. In the case when $a(x)$, $k(u)$, $b(t)$ and $f(x)$ have power lower estimates near the origin, lower power type bounds for solutions $\varphi(x)$ are investigated. Conditions for the uniqueness of the solution in a weighted space of continuous functions are also proved. Particular cases of the equation are specially considered.

1. Introduction. We consider the Volterra nonlinear integral equation of the form

$$\varphi^m(x) = a(x)\int_0^x k(x-t)b(t)\varphi(t)\,dt + f(x), \quad 0 < x < d \leq \infty \tag{1.1}$$

with $m > 0$ and real-valued functions $a(x)$, $k(u)$, $b(t)$ and $f(x)$. This equation generalizes equations investigated by many authors. The equation

$$\varphi^m(x) = \int_0^x k(x-t)\varphi(t)\,dt + f(x), \quad 0 < x < d \leq \infty \tag{1.2}$$

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