

ON A BOUNDARY INTEGRAL METHOD FOR THE  
SOLUTION OF THE HEAT EQUATION IN UNBOUNDED  
DOMAINS WITH A NONSMOOTH BOUNDARY

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ABSTRACT. We study a boundary integral method for the solution of the heat equation in an unbounded domain  $D$  in  $\mathbf{R}^2$ . It is assumed that the boundary of  $D$  is a polygon  $\Gamma = \partial D$  and that  $\mathbf{R}^2 \setminus D$  is a simply connected domain. We use a method which was proposed by Chapko and Kress [2] for the case of a smooth bounded domain  $D$  and analyze this method in the presence of a boundary with corners.

**1. Introduction.** In this paper we study the numerical solution of the following initial value problem

$$(1.1) \quad \begin{cases} u_t(x, t) = c\Delta u(x, t), & (x, t) \in D \times (0, T], \\ u(x, t) = F(x, t), & (x, t) \in \Gamma \times [0, T], \\ u(x, t) \xrightarrow{|x| \rightarrow \infty} 0, & t \in [0, T], \\ u(x, 0) = 0, & x \in D. \end{cases}$$

Here  $D \subset \mathbf{R}^2$  is an unbounded domain and the boundary  $\Gamma := \partial D$  is a polygon. We further assume that the constants  $c$  and  $T$  are greater than zero. The function  $F$  on the boundary should be sufficiently smooth (see Section 2) and should fulfill certain conditions at time  $t = 0$ , especially

$$(1.2) \quad F(\cdot, 0) \equiv 0.$$

There are several ways to approximate the solution of (1.1) with the help of a boundary integral equation. One way would be to use a single

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