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ASYMPTOTIC ERROR ANALYSIS OF A QUADRATURE METHOD FOR INTEGRAL EQUATIONS WITH GREEN'S FUNCTION KERNELS

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ABSTRACT. We conduct an asymptotic error analysis of the trapezoidal quadrature method applied to nonlinear integral equations with Green's function kernels and, based on the asymptotic error expansion of the approximate solution, justify the Richardson extrapolation method. Following the complete error analysis, numerical examples are given to demonstrate the theory. The examples are taken from two-point boundary value problems governed by nonlinear ordinary differential equations, which can be transformed into nonlinear integral equations by using Green's functions. One of the examples involves a regular singular operator for which other well-known numerical techniques such as finite differences may not be applicable.

1. Introduction. Consider nonlinear integral equations in the form

$$u(s) - \int_0^1 G(s,t)\psi\left(t, u(t)\right) dt \;=\; f(s), \quad s \in [0,1], \tag{1.1}$$

where ψ and f are given functions and u is the unknown to be determined. In this paper, we consider *Green's function* type kernels $G(s,t)$ for equation (1.1). Specifically, we let m be a positive integer and assume that $G(s,t)$ satisfies the conditions

(1) $G(s,t) \in C([0,1] \times [0,1])$

(2) $G(s,t) \in C^{2m}((0,1) \times (0,1) \setminus D)$ where

$$D := \{(s,t) \in (0,1) \times (0,1) : s = t\}$$

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