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A NYSTRÖM METHOD FOR A CLASS OF INTEGRAL EQUATIONS ON THE REAL LINE WITH APPLICATIONS TO SCATTERING BY DIFFRACTION GRATINGS AND ROUGH SURFACES

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ABSTRACT. We propose a Nyström/product integration method for a class of second kind integral equations on the real line which arise in problems of two-dimensional scalar and elastic wave scattering by unbounded surfaces. Stability and convergence of the method is established with convergence rates dependent on the smoothness of components of the kernel. The method is applied to the problem of acoustic scattering by a sound soft one-dimensional surface which is the graph of a function f, and superalgebraic convergence is established in the case when f is infinitely smooth. Numerical results are presented illustrating this behavior for the case when f is periodic (the diffraction grating case). The Nyström method for this problem is stable and convergent uniformly with respect to the period of the grating, in contrast to standard integral equation methods for diffraction gratings which fail at a countable set of grating periods.

1. Introduction. The most general form of a Fredholm integral equation of the second kind on the real line is

$$x(s) = y(s) + \int_{-\infty}^{+\infty} k(s,t)x(t)\,dt, \quad s \in \mathbf{R}, \tag{1.1}$$

where the kernel k and the righthand side y are given and the unknown function x is to be determined. In this paper we consider the case when the kernel $k(s,t)$ takes the form

$$k(s,t) = a^*(s,t)\ln|s-t| + b^*(s,t), \tag{1.2}$$

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