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POSITIVE SOLUTIONS OF SINGULAR INTEGRAL EQUATIONS

MARIA MEEHAN AND DONAL O'REGAN

ABSTRACT. Continuous, positive solutions of singular integral equations of the form $y(t) = h(t) + \int_0^T k(t,s)\,[f(y(s)) + g(y(s))]\,ds$ are sought. Here $f : [0,\infty) \to [0,\infty)$ is continuous and nondecreasing while $g : (0,\infty) \to [0,\infty)$ is continuous, nonincreasing and possibly singular. The case when $T = \infty$ is also discussed.

1. Introduction. In the first half of this paper, Schauder's fixed point theorem is used to obtain the existence of continuous, positive solutions of

$$(1.1) \qquad y(t) = h(t) + \int_0^T k(t,s)[f(y(s)) + g(y(s))]\,ds, \quad t \in [0,T].$$

It is assumed that $f : [0,\infty) \to [0,\infty)$ is continuous and nondecreasing, while $g : (0,\infty) \to [0,\infty)$ is continuous, nonincreasing and possibly singular, that is, the possibility of $g(0)$ being undefined is allowed. In Section 2, by placing appropriate conditions on h, k, f and g, we use Schauder's fixed point theorem to prove the existence of a solution $y \in C[0,T]$ such that $0 < \beta < y(t) < \alpha$, $t \in [0,T]$ for some $0 < \beta < \alpha$. In addition a special case of this result, which occurs when $h \in C[0,T]$ is such that $h(t) > 0$, $t \in [0,T]$, is stated for completeness.

In Section 3 we extend the results of Section 2 and consider the possibly singular equation

$$(1.2) \qquad y(t) = h(t) + \int_0^\infty k(t,s)[f(y(s)) + g(y(s))]\,ds, \quad t \in [0,\infty).$$

Schauder's fixed point theorem and the Schauder-Tychonoff fixed point theorem are used to establish the existence of a positive solution $y \in C_l[0,\infty)$ and $y \in BC[0,\infty) \subset C[0,\infty)$ respectively of (1.2). (Here

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