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ANALYSIS AND NUMERICS OF AN INTEGRAL EQUATION MODEL FOR SLENDER BODIES IN LOW REYNOLDS-NUMBER FLOWS

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ABSTRACT. The interaction of particular slender bodies with low Reynolds-number flows is in the limit "slenderness to zero" described by the linear Fredholm integral equation of the second kind

$$c\phi(s) = F(s) + \int_{-1}^{1} \frac{\phi(t) - \phi(s)}{|t-s|}\, dt, \quad s \in [-1, 1],$$

where c is a real number, F is a continuous function and ϕ is unknown. The integral operator T of this equation is symmetric on certain subsets of its domain. T has a denumerable set of eigenvalues of logarithmic growth. The respective eigenspaces contain the Legendre-polynomials. A theorem similar to a classical result of Plemelj-Privalov for integral operators with Cauchy kernels is proved. In contrast to Cauchy kernel operators, T maps no α-Hölder space into itself. A spectral analysis of the restriction $\tilde{T}$ of T to the space of all polynomials is performed. $\tilde{T}$ has a self-adjoint extension $\tilde{T}^{sa}$ in $\mathcal{L}^2([-1,1])$. The spectrum of $\tilde{T}^{sa}$ is a pure point spectrum. The respective eigenspaces are spanned by Legendre-polynomials. A spectral method based on expansions in terms of the Legendre polynomials is presented and stability and convergence properties are proved. The results are illustrated by several numerical simulations. In case of sufficiently smooth functions F a modified spectral method is proposed. For that method uniform stability and convergence results are proved.

1. Introduction. The starting point of the subsequent investigations is a model for the shape of a long, slender body (e.g., a fiber) exposed to a normal flow [**6**]. Applying the singularity method for linearized fluid dynamics [**1**], [**15**] and under several assumptions (fluid velocity approaches a constant value as the spatial variable tends to

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