JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 12, Number 2, Summer 2000

ON THE RANGE OF THE STRUVE $\mathbf{H}_\nu$-TRANSFORM

VU KIM TUAN

ABSTRACT. The range of the $\mathbf{H}_\nu$-transform on some spaces of functions is described.

1. Introduction. The Struve $\mathbf{H}_\nu$-transform as an example of an asymmetric Watson transform is defined as [**8**], [**9**]

$$f(x) = (\mathbf{H}_\nu g)(x) = \int_0^\infty \sqrt{xy}\mathbf{H}_\nu(xy)g(y)\,dy, \quad x \in (0,\infty) = R_+, \tag{1}$$

if the integral converges in some sense (absolutely, improper or mean convergence). Here $\mathbf{H}_\nu(x)$ is the Struve function [**1**]. The boundedness and range of the Struve $\mathbf{H}_\nu$-transform on the space $\mathcal{L}_{\mu,p}$ of functions f, measurable on R_+, and such that

$$\|f\|_{\mu,p} = \left\{\int_0^\infty |x^\mu f(x)|^p \frac{dx}{x}\right\}^{1/p} < \infty, \quad 1 \le p < \infty, \tag{2}$$

have been considered in [**2**], [**4**], [**5**]. It has been proved there that, under some restrictions on parameters ν, μ, p, the range of the Struve $\mathbf{H}_\nu$-transform (1) coincides with the range of the Hankel transform

$$f(x) = (\mathcal{H}_{\nu+1} g)(x) = \int_0^\infty \sqrt{xy} J_{\nu+1}(xy) g(y)\,dy, \quad x \in R_+, \tag{3}$$

on the space $\mathcal{L}_{\mu,p}$. It is well known that the Hankel transform (3) is an automorphism on the space $L_2(R_+) = \mathcal{L}_{1/2,2}$, hence in the strip $-2 < \mathcal{R}e\,\nu < 0$ the Struve $\mathbf{H}_\nu$-transform is bounded on $L_2(R_+)$, and moreover, if $\mathcal{R}e\,\nu \neq -1$, its range is the whole space $L_2(R_+)$:

$$\|\mathbf{H}_\nu g\|_{L_2(R_+)} \le C\|g\|_{L_2(R_+)}, \quad -2 < \mathcal{R}e\,\nu < 0, \tag{4}$$

$$\|g\|_{L_2(R_+)} \le C\|\mathbf{H}_\nu g\|_{L_2(R_+)}, \quad |1 + \mathcal{R}e\,\nu| < 1, \tag{5}$$

Received by the editors on August 1, 1996.
Key words and phrases. $\mathbf{H}_\nu$- and Y_ν-transforms, range of integral transforms.
AMS *Mathematics Subject Classification.* 44.
Supported by the Kuwait University research grant SM 112.