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OVERLAPPING ADDITIVE SCHWARZ PRECONDITIONERS FOR BOUNDARY ELEMENT METHODS

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Dedicated to Prof. Ian H. Sloan on the occasion of his 60th birthday

ABSTRACT. We study overlapping additive Schwarz preconditioners for the Galerkin boundary element method when used to solve Neumann problems for the Laplacian. Both the h and p versions of the Galerkin scheme are considered. We prove that the condition number of the additive Schwarz operator is bounded by $O(1+\log^2(H/\delta))$ for the h version, where H is the size of the coarse mesh and δ is the size of the overlap, and bounded independently of the mesh size and the polynomial order for the p version.

1. Introduction. We consider in this paper the Neumann problem for the Laplace equation in the exterior of a curve in the plane. Via the standard fundamental solution, we reformulate the problem as a boundary integral equation of the first kind with a symmetric kernel. The Galerkin method when used to solve this equation results in solving a linear system, the coefficient matrix $\mathcal{A}$ of which is *dense*. If N is the size of $\mathcal{A}$, then the Gauss solver requires $O(N^3)$ operations for computation of the coefficients giving the corresponding Galerkin approximate solution. Hence when N is large, one resorts to iterative methods. The matrix being symmetric and positive definite, the conjugate gradient method is among the most practical and efficient iterative methods. Since $\mathcal{A}$ is ill-conditioned, in the sense that its condition number increases with N, the convergence rate of the conjugate gradient method will deteriorate, which leads to a huge number of iterations needed to achieve accuracy.

Several authors have investigated preconditioners for this equation. Multigrid preconditioners, wavelet approximation and matrix com-

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