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A NOTE ON THE SOLUTION SET OF INTEGRAL INCLUSIONS

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ABSTRACT. In this note we discuss the topological structure of the set of solutions of integral and differential inclusions.

1. Introduction. This paper discusses the structure of the solution set of the Volterra integral inclusion

$$y(t) \in h(t) + \int_0^t k(t,s)F(s,y(s))\,ds \quad \text{for } t \in [0,T]. \tag{1.1}$$

Throughout $k : [0,T] \times [0,t] \to \mathbf{R}$ and $F : [0,T] \times \mathbf{R}^n \to CK(\mathbf{R}^n)$; here $CK(\mathbf{R}^n)$ denotes the family of all nonempty, compact, convex subsets of $\mathbf{R}^n$. In the literature only a few results have appeared on the structure of the solution set of (1.1); we refer the reader to [**1**, p. 219] and the references therein. For completeness we state here the main result available in the literature [**1**]. Let $S(h;\mathbf{R}^n)$ denote the solution set of (1.1).

Theorem 1.1. *Let* $k : [0,T] \times [0,t] \to \mathbf{R}$, $F : [0,T] \times \mathbf{R}^n \to CK(\mathbf{R}^n)$ *and suppose the following conditions hold*:

$$t \longmapsto F(t,x) \quad \textit{is measurable for every } x \in \mathbf{R}^n \tag{1.2}$$

$$\begin{cases} x \longmapsto F(t,x) \quad \textit{is upper semicontinuous } (u.s.c.) \\ \textit{for a.e. } t \in [0,T] \end{cases} \tag{1.3}$$

$$\begin{cases} \textit{there exists } h \in L^1[0,T] \textit{ with } \|F(t,x)\| \le h(t) \\ \textit{for a.e. } t \in [0,T] \textit{ and every } x \in R^n \end{cases} \tag{1.4}$$

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