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AN EXPLICIT BOUNDARY INTEGRAL REPRESENTATION OF THE SOLUTION OF THE TWO-DIMENSIONAL HEAT EQUATION AND ITS DISCRETIZATION

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ABSTRACT. An explicit representation of the solution of the two-dimensional heat equation through solutions of boundary integral equations is given. Using this representation we get a semi-discretization, in time, where a sequence of boundary integral equations must be solved. For the last ones the collocation quadrature method can be used.

1. Introduction. The horizontal line method, Rothe's method, is often used in order to solve the heat equation numerically. This method consists of a time discretization by finite differences and leads to a sequence of boundary value problems for an inhomogeneous elliptic equation. The last ones can be solved using finite element, finite difference or other methods. In order to overcome difficulties caused by a complicated geometry of the problem the method of integral equations is often used where volume potentials due to the inhomogeneity are incorporated. The advantages of the boundary integral equation method, such as the dimension reduction, the simplification of the geometry of the problem in two-dimensional case, possibility of solving of exterior problems, etc., can be preserved by using approaches involving only boundary integrals, see [**3, 8**] and references therein, which leads to very effective discretizations especially in cases of a complicated geometry. The kernels of the boundary integral equations in [**8**] are evaluated by the Cauchy integral formula and numerical integration in the complex plane. A more explicit procedure for the computation of the kernels involved in the sequence of boundary integral equations and based on some recurrence equations is given in [**3**]. A drawback of the line method is a fixed convergence order independent of the smoothness of the initial data.

The aims of this paper are to overcome this difficulty and to give an explicit representation of the solution of the two-dimensional heat

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