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CONDITIONAL STABILIZING ESTIMATION FOR AN INTEGRAL EQUATION OF FIRST KIND WITH ANALYTIC KERNEL

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Dedicated to the memory of
Professor Dr. rer. nat. habil. Siegfried Prößdorf

ABSTRACT. We discuss an integral equation of first kind which arises from an inverse problem for detecting steel reinforcement bars. Since the kernel of this integral equation is analytic, our problem is severely ill-posed. We transform the problem for this integral equation into a Cauchy problem for Laplace's equation. By using the conditional estimation of the Cauchy problem for the elliptic operator, we prove that, under suitable hypotheses, a logarithmic stabilizing estimation holds for the solution of the integral equation. In addition, this method can be used to determine discontinuous points of the solution of the integral equation.

1. Introduction. In [**3**], Engl and Isakov discuss an inverse problem for detecting steel reinforcement bars inside of concrete. Under suitable hypotheses, they transform the problem into a first kind integral equation with analytic kernel and prove the uniqueness of the solution of the integral equation. From the theory of integral operators, we know this integral equation is a severely ill-posed problem, since the singular values of the integral operator decrease rapidly, e.g., [**5, 13**]. This kind of integral equation is also proposed in geophysics, e.g., Ramm [**9, 10**]. The same integral equation is discussed by Lavrent'ev [**6**], Bukhgeim [**1**] and Serikbaev [**11**].

In this paper, we transform the integral equation into a Cauchy problem for Laplace's equation, which has been studied extensively.

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