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PARTIALLY-COUPLED INTEGRAL EQUATIONS FOR A DYNAMIC FRACTURE PROBLEM IN COUPLED THERMOELASTICITY

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ABSTRACT. Robust asymptotic solution forms reduce a canonical problem of dynamic fracture in a thermoelastic body to a set of partially-coupled integral equations. The set contains both Cauchy and Abel operators but can be solved analytically. The solution shows aspects of the effects of thermoelastic coupling.

1. Introduction. When crack growth is rapid, fracture is a dynamic process and, in a linear thermoelastic solid [**9**], is governed by a fully-coupled system of temperature and linear momentum equations. The growth of a crack of infinite width and semi-infinite length in an unbounded solid is a canonical problem of dynamic fracture in plane strain [**2, 13**], and a simple version suitable for coupled thermoelasticity is sub-critical, steady-state growth driven by forces and heat fluxes applied to opposite faces of the crack as line loads. The line loads lie parallel to the crack edge, and are moved behind it at a fixed distance.

2. Governing equations. In the steady-state, crack growth is at a constant speed, and field variables depend explicitly only on the spatial coordinates $\mathbf{x}$ moving with the crack. If $\mathbf{x}$ is taken as the Cartesian system $\mathbf{x} = (x, y, z)$ affixed so that $(y = 0, x > 0)$ always defines the crack edge and growth is in the negative-x direction, the equations of thermoelasticity become [**4, 6**]

(2.1a)
$$\nabla^2\mathbf{u} + (m^2-1)\nabla\Delta + \chi\nabla\theta - m^2c^2\frac{\partial^2\mathbf{u}}{\partial x^2} = 0, \quad \chi = \chi_0(4-3m^2)$$

(2.1b)
$$h\nabla^2\theta - c\frac{\partial}{\partial x}\left(\theta - \frac{m^2\varepsilon}{\chi}\Delta\right) = 0$$

(2.1c)
$$\frac{1}{\mu}\,\sigma = [(m^2-2)\Delta + \chi\theta]\,\mathbf{I} + \nabla\mathbf{u} + \mathbf{u}\nabla$$

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