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SOLUTIONS OF INTEGRO-DIFFERENTIAL EQUATIONS ON THE HALF-AXIS WITH RAPIDLY DECREASING NON-DIFFERENCE KERNELS

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ABSTRACT. The purpose of this paper is to investigate the set of all solutions of the integro-differential equation (1) and to obtain a convenient algorithm for calculation of any solution. Both objectives are obtained in the case when the integral kernel $R_1(x)$ is even and both kernels $R_1(x)$ and $R_2(x)$ in the equation rapidly decrease as x approaches infinity, although the integrals

$$\phi_i(0) \equiv \int_{-\infty}^{\infty} R_i(x)\, dx, \quad i = 1, 2$$

are not assumed to be small. To be sure that integrals in the equation converge, the sought for solutions are supposed to satisfy a condition of the type:

$$|y(x)| < \text{const} \cdot e^{\lambda x}.$$

The asymptotic behavior of solutions as $x \to \infty$ is defined by the number $\phi_1(0)$. If $\phi_1(0) < 1$, then there is a positive number $p^\star$ such that all solutions grow proportionally to $e^{p^\star x}$ except specific ones which tend to zero as $e^{-p^\star x}$. If $\phi_1(0) = 1$, then all solutions grow as linear functions except the specific ones which tend to a constant as $x \to \infty$. If $\phi_1(0) > 1$, there exists a purely imaginary number $p^\star$ such that the asymptotic behavior of solutions as $x \to \infty$ is described by an oscillating function which is a linear combination of two specific solutions which behave as $e^{i|p^\star|x}$ and $e^{-i|p^\star|x}$, respectively.

In all these cases the condition

$$y'(0) = \mu y(0)$$

is found which ensures a solution to be specific.

In many physical applications involving the considered problem it is the coefficient μ which is important. To evaluate the parameter μ an asymptotic series convergent to μ is found.

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