

**FACTORIZATION OF SINGULAR INTEGRAL  
OPERATORS WITH A CARLEMAN SHIFT  
AND SPECTRAL PROBLEMS**

V.G. KRAVCHENKO, A.B. LEBRE AND J.S. RODRÍGUEZ

*Dedicated to Professor G.S. Litvinchuk  
on the occasion of his seventieth birthday*

**ABSTRACT.** In this paper we study singular integral operators with a linear fractional Carleman shift preserving the orientation on the unit circle. The main goal is to characterize the spectrum of some of these operators. To this end a special factorization of the operator is derived with the help of a factorization of a matrix function in a suitable algebra. After developing methods which permit us to obtain a factorization of a matrix function for some classes of interest, the spectral analysis of some types of singular integral operators with a Carleman shift is done.

**1. Introduction.** In this paper we study singular integral operators with a Carleman shift on the unit circle  $\mathbf{T}$  of a special nature, the so-called linear fractional Carleman shift preserving the orientation of  $\mathbf{T}$  (see (1.1)). The main objective is to develop tools which permit us to give a complete description of the spectrum for some classes of these operators acting in the Lebesgue space  $L_p(\mathbf{T})$ ,  $p \in (1, \infty)$ . This includes the identification of the Fredholm and non-Fredholm parts of the spectrum of such an operator  $T$  (the latter, in general, is known for the type of operators which we are going to consider here), as well as the calculation of the defect numbers of the operator  $T - \lambda I$ , for all  $\lambda$  belonging to the Fredholm part of the spectrum of  $T$ , and a representation for the resolvent operator. In this respect, this paper may be seen as a continuation of [7], where the spectral analysis was made for a very special type of operators.

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