ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 43, Number 5, 2013

ON THE PATHWISE UNIQUENESS OF STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS WITH NON-LIPSCHITZ COEFFICIENTS

DEFEI ZHANG AND PING HE

ABSTRACT. In this paper, we prove a pathwise uniqueness result of a class of stochastic partial differential equations driven by space-time white noise whose coefficients satisfy non-Lipschitz conditions.

1. Introduction. Many mathematicians and physicists have investigated the uniqueness of the following stochastic partial differential equations (SPDE):

$$\frac{\partial}{\partial t}\nu_t(x) = \Delta\nu_t(x) + \sigma(\nu_t(x))\underline{\dot{W}}(t,x), \quad \nu_0 = \mu, \tag{1.1}$$

where $\underline{\dot{W}}$ is the space-time white noise. It is a very important model which was proposed by Dawson in 1972 as follows:

$$\frac{\partial}{\partial t}\nu_t(x) = \Delta\nu_t(x) + \sigma\sqrt{\nu_t(x)}\underline{\dot{W}}(t,x), \quad \nu_0 = \mu. \tag{1.2}$$

In this case, the uniqueness of the solution of the SPDE (1.2) is only proved in the weak sense using that of the martingale problem. The difficulty in proving pathwise uniqueness in (1.2) arises from the fact that $\sqrt{\nu(t,x)}$ is non-Lipschitz. For a more detailed description the

2010 AMS *Mathematics subject classification.* Primary 60H10, 60H20.
Keywords and phrases. Backward doubly stochastic differential equations, stochastic partial differential equations, pathwise uniqueness, non-Lipschitz coefficients.
This work was supported by The National Basic Research Program of China (973 Program) (No. 2007CB814901) and The Scientific Research Foundation of Yunnan Province Education Committee (No. 2010Y167), National Science Foundation of China (No. 11301160), Natural Science Foundation of Yunnan Province (No. 2013FZ116) and Scientific Research Foundation of Yunnan Province Education Committee (No. 2011C120), National Science Foundation of China grant 11301160, Natural Science Foundation of Yunnan Province (No. 2013FZ116) and Scientific Research Foundation of Yunnan Province Education Committee (No. 2011C120).
Received by the editors on December 10, 2010, and in revised form on March 6, 2011.
DOI:10.1216/RMJ-2013-43-5-1739