ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 43, Number 5, 2013

PERFECT QUADRILATERAL RIGHT PRISMS

ALLAN J. MACLEOD

ABSTRACT. We consider right prisms with horizontal quadrilateral bases and tops, and vertical rectangular sides. We look for examples where all the edges, face diagonals and space diagonals are integers. We find examples when the base is an isosceles trapezium or a parallelogram, but no solution for a kite or rhombus.

1. Introduction. Let **ABCD** be a convex quadrilateral in the x-y plane, and let **EFGH** be the identical shape vertically translated h units. Then the three-dimensional object with vertices **ABCDEFGH** is known as a right prism. In this paper, we consider finding such prisms where the lengths of the edges, face diagonals and space diagonals are all integers.

If **ABCD** and **EFGH** are rectangles, then the prism is in fact a cuboid, and the problem is that of finding a perfect rational cuboid, which is still a classic unsolved problem, see Bremner [**1**], Dickson [**3**], Guy [**4**], Leech [**6**] and van Luijk [**7**]. Very recently, Sawyer and Reiter [**11**] have shown the existence of perfect parallelipipeds. These are not normally right prisms, in that the solutions found do not have rectangular vertical faces.

In this paper, we consider quadrilaterals such as the trapezium, the parallelogram, the kite and the rhombus, to see whether perfect right prisms with these base shapes exist. It is interesting to note that a mathematician of the caliber of Kummer [**5**] considered the problem of quadrilaterals with integer sides and diagonals.

For the aid of readers, we summarize the results as:

Theorem. *Perfect right prisms exist when the base is an isosceles trapezium or parallelogram. We conjecture that there are an infinite number of non-congruent isosceles trapezia that lead to perfect prisms.*

Received by the editors on June 14. 2010, and in revised form on February 16, 2011.

DOI:10.1216/RMJ-2013-43-5-1689