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EXISTENCE OF NONOSCILLATORY SOLUTIONS TO SECOND-ORDER NONLINEAR NEUTRAL DYNAMIC EQUATIONS ON TIME SCALES

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ABSTRACT. By employing Kranoselskii's fixed point theorem, we establish the existence of nonoscillatory solutions to the second-order nonlinear neutral dynamic equation $[r(t)(x(t)+p(t)x(g(t)))^{\Delta}]^{\Delta} + f(t, x(h(t))) = 0$ on a time scale. In particular, one interesting example is included to illustrate the versatility of our results.

1. Introduction. Consider second-order nonlinear neutral dynamic equations of the form

$$[r(t)(x(t) + p(t)x(g(t)))^{\Delta}]^{\Delta} + f(t, x(h(t))) = 0 \tag{1}$$

on a time scale $\mathbf{T}$. The motivation originates from [**6, 8**], where some open problems were presented in [**6**] and some conditions for the existence of nonoscillatory solutions of first-order nonlinear neutral dynamic equation $[x(t)+p(t)x(g(t))]^{\Delta}+f(t, x(h(t))) = 0$ were presented in [**8**]. In this paper, by employing Kranoselskii's fixed point theorem, we try to find some conditions for the existence of nonoscillatory solutions of (1). We remark that there has been a number of researchers studying oscillatory behaviors for dynamic equations on time scales, see, e.g., [**1–3, 5–7**] and the references therein. However, there are few papers discussing the existence of nonoscillatory solutions for neutral functional dynamic equations on time scales. For a neutral functional dynamic equation, the highest derivative of the unknown function appears with the argument t (the present state of the system) as well as one or more deviating arguments (the past or future state of the system).

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