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MINIMAL PRIME IDEALS AND CYCLES IN ANNIHILATING-IDEAL GRAPHS

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ABSTRACT. Let R be a commutative ring with identity, and let $\mathbf{A}(R)$ be the set of ideals with non-zero annihilator. The *annihilating-ideal graph* of R is defined as the graph $\mathbf{AG}(R)$ with the vertex set $\mathbf{A}(R)^* = \mathbf{A}(R) \setminus \{0\}$, and two distinct vertices I and J are adjacent if and only if $IJ = 0$. In this paper, we study some connections between the graph theoretic properties of this graph and some algebraic properties of a commutative ring. We prove that if $\mathbf{AG}(R)$ is a tree, then either $\mathbf{AG}(R)$ is a star graph or a path of order 4 and, in the latter case, $R \cong F \times S$, where F is a field and S is a ring with a unique non-trivial ideal. Moreover, we prove that if R has at least three minimal prime ideals, then $\mathbf{AG}(R)$ is not a tree. It is shown that, for every reduced ring R, if R has at least three minimal prime ideals, then $\mathbf{AG}(R)$ contains a triangle. Also, we prove that, for every non-reduced ring R, if $|\mathrm{Min}\,(R)| = 2$, then either $\mathbf{AG}(R)$ contains a cycle or $\mathbf{AG}(R) \cong P_4$. Finally, it is proved that, if $|\mathrm{Min}\,(R)| = 1$ and $\mathbf{AG}(R)$ is a bipartite graph, then $\mathbf{AG}(R)$ is a star graph.

1. Introduction. The study of algebraic structures, using the properties of graphs, became an exciting research topic in the past 20 years, leading to many fascinating results and questions. There are many papers on assigning a graph to a ring, for instance, see [**1–4, 7, 10, 11, 14**]. Throughout this paper, all rings are assumed to be non-domain commutative rings with identity. A *multiplicative closed subset* of a commutative ring R is a subset S of R such that $1 \in S$ and, for every $x, y \in S$, $xy \in S$. By $\mathrm{Min}\,(R)$ and $Z(R)$, we denote the set of all minimal prime ideals of R and the set of all zero-divisors of R, respectively. A ring R is said to be *reduced*, if it has no non-zero

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