ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 43, Number 4, 2013

TWIST POINTS OF A JORDAN DOMAIN

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1. Introduction. In this paper we will describe how the twist points of a Jordan domain are distributed about each other. Our description will indicate in what sense Ostrowski's condition fails at a twist point. We first introduce the background material and notation. Many of the definitions are found in McMillan's papers.

Let D be a bounded Jordan domain and J its boundary. On $D \cup J$ we define the relative distance d_D between two points as the infimum of the Euclidean diameters of curves lying in D and joining these two points. Any limits involving boundary points will be with respect to the metric, d_D.

Let $f(z)$ be a one-to-one conformal map of the unit disk onto D. It is well known that $f(z)$ can be extended to a homeomorphism of the closed unit disk onto $D \cup J$. A subset $N \subset J$ is said to be a D-conformal null set if $\{e^{i\theta} : f(e^{i\theta}) \in N\}$ has measure zero. This definition is independent of f. Let $T \subset J$ denote the set of points where the inner tangent to J exists. That is, if $a \in T$, then there is a unique $v(a)$, $0 \le v(a) < 2\pi$, such that, for each $\varepsilon > 0$, $\varepsilon < \pi/2$, there exists a $\delta > 0$ such that

$$\Delta = \{a + \rho e^{i\varphi} : 0 < \rho < \delta,\ |\varphi - v(a)| < \pi/2 - \varepsilon\} \subset D,$$
$$\text{and } d_D(w, a) \to 0 \text{ as } |w - a| \to 0,\ w \in \Delta.$$

Let R be those $a \in J$ such that

$$\liminf_{\substack{w \to a \\ w \in D}} \arg(w - a) = -\infty \quad \text{and} \quad \limsup_{\substack{w \to a \\ w \in D}} \arg(w - a) = +\infty,$$

where $\arg(w - a)$ is defined and continuous in D. It has been shown [**4**, page 44] that $J = T \cup R \cup N$, where N is a D-conformal null set. There are examples of domains D such that $J = R \cup N$. See [**4**, pages 65–67] and [**6**, pages 736–738]. The set R is called the set of twist points of D.

2010 AMS *Mathematics subject classification.* Primary 30D40, 30C35.
Received by the editors on August 22, 2008, and in revised form on September 26, 2008.
DOI:10.1216/RMJ-2013-43-4-1317