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EXISTENCE AND UNIQUENESS OF POSITIVE SOLUTIONS FOR FOURTH-ORDER m-POINT BOUNDARY VALUE PROBLEMS WITH TWO PARAMETERS

XINAN HAO, NAIWEI XU AND LISHAN LIU

ABSTRACT. This paper deals with the existence and uniqueness of positive solutions to fourth-order m-point boundary value problems with two parameters. The arguments are based upon a specially constructed cone and a fixed point theorem in a cone for a completely continuous operator, due to Krasnoselskii and Zabreiko. The results obtained herein generalize and complement the main results of [**7, 10**].

1. Introduction. In this paper, we will study the existence and uniqueness of positive solutions for the following fourth-order m-point boundary value problems with two parameters

$$\begin{cases} u^{(4)} + \alpha u'' - \beta u = f(t,u), \quad 0 < t < 1, \\ u(0) = \displaystyle\sum_{i=1}^{m-2} a_i u(\xi_i), \ u(1) = \sum_{i=1}^{m-2} b_i u(\xi_i), \\ u''(0) = \displaystyle\sum_{i=1}^{m-2} a_i u''(\xi_i), \ u''(1) = \sum_{i=1}^{m-2} b_i u''(\xi_i), \end{cases} \tag{1.1}$$

where $\alpha,\ \beta \in \mathbf{R}$, $\xi_i \in (0,1)$, $a_i, b_i \in \mathbf{R}^+$ for $i \in \{1,2,\ldots,m-2\}$ are given constants, $\mathbf{R}^+ = [0,+\infty)$, and $\alpha,\ \beta,\ f$ satisfy

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The first author is the corresponding author.
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