

## MULTIPLIERS ON $L^p$ -SPACES FOR HYPERGROUPS

SINA DEGENFELD-SCHONBURG AND RUPERT LASSER

**ABSTRACT.** Let  $K$  be a commutative hypergroup. At first, we characterize the space of multipliers on  $L^p(K, m)$ . Then, we investigate the multipliers on  $L^1(\mathcal{S}, \pi)$  and  $L^2(\mathcal{S}, \pi)$ , where  $\mathcal{S}$  is the dual space of  $K$ , i.e.,  $\mathcal{S} = \text{supp } \pi$ ,  $\pi$  is the Plancherel measure of  $K$ .

**1. Introduction.** There are a lot of results on multipliers defined on translation-invariant Banach spaces on a locally compact group  $G$ . The standard reference for that is the book by Larsen [6]. In this paper we investigate multipliers in the hypergroup setting. Hypergroups generalize locally compact groups. For the theory of hypergroups, we refer to [1, 5]. A hypergroup  $K$  is a locally compact Hausdorff space with a convolution, i.e., a map  $K \times K \rightarrow M^1(K)$ ,  $(x, y) \mapsto \delta_x * \delta_y$ , ( $M^1(K)$  is the space of probability measures on  $K$ ) and an involution, i.e.,  $K \rightarrow K$ ,  $x \mapsto \tilde{x}$ , satisfying certain axioms, see [1].

Many results of harmonic analysis can be shown for hypergroups, in particular, for commutative hypergroups. In the following, we assume throughout that  $K$  is a commutative hypergroup. For a locally compact Hausdorff space  $X$  let  $C(X)$ ,  $C^b(X)$ ,  $C_0(X)$  and  $C_{00}(X)$  be the spaces of all continuous functions on  $X$ , those that are bounded, those that vanish at infinity and those that have compact support.  $M(X)$  denotes the space of all regular complex Borel measures on  $X$  which can be identified with  $C_0(X)^*$ , the dual space of  $C_0(X)$ .

The convolution allows us to define a translation operator on  $C(K)$  by setting

$$T_x f(y) = \int_K f(z) d(\delta_x * \delta_y)(z)$$

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