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SPECTRAL PROPERTIES OF THE SIMPLE LAYER POTENTIAL TYPE OPERATORS

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ABSTRACT. We establish the exact asymptotical behavior of singular values of the simple layer potential type operators.

1. Introduction and notation. It is well known that any function $f \in C^2(\overline{\Omega})$ $(\Omega \subset \mathbf{R}^n, n \geq 2)$ may be expressed as

$$f(x) = \int_\Omega u(x-y) \triangle f(y)\, dy + \int_{\partial\Omega} f(y) \frac{\partial u(x-y)}{\partial n_y}\, dS_y - \int_{\partial\Omega} \frac{\partial f}{\partial n_y} u(x-y)\, dS_y \quad (x \in \Omega),$$

where

$$u(x) = \begin{cases} -1/(n-2)\,\sigma_n\, |x|^{n-2}, & n > 2 \\ -1/2\pi \ln 1/|x|, & n = 2 \end{cases}$$

and $\sigma_n = 2\pi^{n/2}/\Gamma(n/2)$ is the area of $(n-1)$-dimensional sphere.

Here, Ω is a domain in $\mathbf{R}^n$, $\triangle$ is the Laplace operator, $\partial/\partial n_y$ denotes the derivative in the direction of external normal to $\partial\Omega$ with respect to y and dS_y denotes the area element.

Operators

$$g \longmapsto \int_\Omega u(x-y)\, g(y)\, dy,$$

$$g \longmapsto \int_{\partial\Omega} -u(x-y)\, g(y)\, dS_y$$

and

$$g \longmapsto \int_{\partial\Omega} \frac{\partial u(x-y)}{\partial n_y} g(y)\, dS_y$$

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